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L5 SYSTEM:

Role of Computing and Precision Measurements

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The L5 Coaxial-Carrier Transmission System is the first long-haul, high-capacity transmission system for which the design was strongly influenced by extensive application of computer-aided design (CAD) techniques. A tight time schedule required a parallel effort of (i) improving and using the somewhat limited capabilities of existing CAD programs and (ii) developing new programs having increased capability and efficiency. Similar development of computer-controlled measurement techniques provided necessary device and component characterization and subsystem evaluation. The result is a powerful group of tools that are independently important, but whose combined use helped make possible the timely completion of the L5 system design. These tools, now "proved in," will profoundly influence the next system design philosophy.

I. INTRODUCTION

At the beginning of the L5 project in 1968, the set of computer-aided design (CAD) tools and the specialized computer-operated transmission measurement set (COTMS) at the Merrimack Valley location of Bell Laboratories were in an early stage of evolution. The tight system development schedule prevented the thorough, leisurely development of sophisticated programs that could provide the complete analysis and characterization needed for high confidence in design. Rather, a parallel effort was undertaken to augment existing design aids and to develop improved ones. The result was immediate answers generated inefficiently for early designs and a powerful group of programs that

bave been "proved in" by use during the later stages of L5 development. Tbese were, however, not completed early enough in tbe design cycle to allow their full impact to be exerted on tbe overall design process in tbe manner that is now possible.

The following sections illustrate typical uses of computer programs for seven pbases of tbe design sequence. Tbese include:

(*i*) Small signal ac analysis of circuits having many nodes, including sensitivity and tolerance analyses.
(*ii*) Optimization of element values in a circuit.
(*iii*) Statistical measurements of early production models of repeaters for use in equalizer design.
(*iv*) Nonlinear distortion analysis.
(*v*) Component characterization.
(*vi*) Overall system analysis.
(*vii*) Equalizer cbaracterization and alignment during manufacturing.

Limitations as well as successes are included to illustrate why and how tbe evolution of programs took place.

II. SMALL-SIGNAL AC CIRCUIT ANALYSIS

Several general-purpose circuit analysis programs,[1] as well as optimization[2] and Monte Carlo[3] programs, were in wide use in tbe communications industry by 1968. However, none was able to handle a circuit of tbe size (over 80 nodes) and complexity of the L5 repeater. While the next generation programs were being developed, modification of tbe existing programs and segmentation of the circuit were necessary to solve the immediate circuit analysis problems.

Since the L5 system uses amplitude modulation, tbe repeater must be a very linear device; this places strong emphasis on accurate, small-signal ac analysis of all circuits in the signal path. Tbe basic repeater amplifier alone contains 80 nodes, 111 passive components, 8 transistors, and 4 wideband transformers. These numbers do not include "parasitic" elements found to be essential for accurate modeling so tbat, in practice, the actual node and component count must be significantly increased.

This section highligbts some of the practical difficulties encountered witb large circuits and briefly describes the next generation program that evolved from tbese experiences.

2.1 Partitioning large networks for analysis

To overcome the size problem, the circuit was at first partitioned into blocks that could be analyzed with existing programs. One output of eacb of tbese analyses was a multiport admittance matrix characterizing tbe block at those nodes where it interconnects with other circuit blocks. These block representations were stored in a library for subsequent retrieval when the performance of a multiblock circuit was to be calculated. This approach of using a frequency-dependent matrix to represent subcircuits proved useful, not only for reducing node and element count, but also for allowing tbe direct use of measured data from COTMS to characterize components not easily or accurately modeled by lumped element values.

Tbe cireuit segmentation approaeh also aided tbe calculation of sensitivities to element values. As eacb element in a given block was varied to determine overall circuit sensitivity to tbat element, the matrix representations of other blocks remained unchanged, saving much calculation time for the overall problem. Sensitivity data proved useful in many ways. Since a full-scale Monte Carlo analysis of tbe circuit was impossible during early development, a linear sensitivity model was used to set component tolerances based on tbeir predicted effects on frequency response. Tbe sensitivity data on parasitic element values also identified critical areas of the circuit where careful modeling and control were essential. It further identified those elements to be subjected to optimization and smaller-scale Monte Carlo analysis.

The segmentation approach also permitted use of the early optimization and Monte Carlo programs.[4] These were particularly belpful in the design of the feedback networks which, thougb small compared to the overall repeater, were critical in determining the frequency shaping of the repeater.

Although the "divide and eonquer" method did allow some success in the early stages of analysis, several diffieulties beeame apparent that were eorrected in tbe next generation programs. The many steps involved in the piecemeal approach led to analysis turnaround times of the order of one day for a complete circuit. This severely limited the number and type of "what if" questions that eould be raised and answered. Not only was tbe overall turnaround time relatively long, but all the segments had to be run in a batcb mode, as no interactive facilities were available for the designer to pose such "what if" questions quickly or effectively. These inefficiencies, coupled with tbe nonstatic nature of the actual circuit as various engineers wanted to

implement changes suggested hy earlier analysis steps, made the turnaround time a serious prohlem. The multiple passes through various analysis segments also led to interfacing problems and increased probability of coding errors.

2.2 A new ac analysis program

The program developed to overcome most previous limitations is called circuit analysis program for efficient computer-optimized design (CAPE-COD). It achieves its power and efficiency through a sparse matrix technique and special, machine-generated coding. It has analyzed circuits with 120 nodes, provides a wide variety of engineering-oriented outputs, allows simple problem formulation by engineers, and provides interactive capahility on a graphics terminal.

The formulation technique chosen for CAPE-COD uses the tableau[5] approach. This method of arranging the system of equations that defines the topology and hehavior of a circuit not only yields an efficient analysis scheme, hut also gives flexihility and simplicity to the program design while minimizing the restrictions placed on the circuits that can be analyzed. For example, the following types of elements that are seldom allowed in ac analysis programs *are* allowed in CAPE-COD: controlled voltage and current sources; zero-valued resistors, capacitors, and inductors; and devices that have no Y or Z matrix representations, but have an S matrix representation (e.g., an ideal transformer or a three-port ideal circulator). By inserting zero-valued components into the original circuit where one suspects parasitics to he important, the program can then systematically investigate their effects without recoding, simply hy varying the values from zero on suhsequent runs.

The elimination order for the tahleau can produce, at one extreme, essentially a nodal formulation; at the other extreme, a mesh formulation; in general, a hyhrid formulation results. This flexihility gives the tableau method the capability of producing more accurate results than other methods. Such accuracy improvements are important when an element or suhcircuit has a Y or Z matrix representation that is ill-conditioned, e.g., the Y matrix of a very small resistance.

To minimize core memory requirements of the computer and to allow analysis of large circuits having over 100 nodes, sparse matrix reduction techniques are used throughout the program. For efficiency in execution time, a loopless machine code program to solve the tahleau system of equations is generated first for each prohlem. This efficiency is most important when the analysis package is used as a

subroutine in an iterative program such as optimization or tolerance analysis, since the time spent generating the machine code is insignificant compared to the time for repeated analyses. In addition to this machine code, dynamic memory storage allocation and memory paging schemes are used to increase efficiency further.

Input data for CAPE-COD can be entered in completely free format, which not only provides great convenience for the user, but also reduces the number of coding errors in large circuits. The input "language" was developed to be natural for an engineer, easily understood and remembered. The CAPE-COD currently accepts the following circuit elements: resistors, capacitors, inductors, controlled or independent voltage or current sources, two forms of transmission lines, two models for transistors (one of which is used for distortion analysis, described later), and N-port black-box devices. The latter can be characterized by directly measured Y, Z, or S parameter data, or by a user-supplied subroutine that calculates these parameters. In addition to having fixed values, elements can be optionally defined as a tabular function of frequency. This black-box capability was essential to the L5 repeater analysis, since no sufficiently accurate lumped element model existed to represent the wideband transformers used in the forward and feedback paths. Instead, two-, three-, and four-port matrix representations were formulated from frequency-dependent measured data

Perhaps the most important improvement over previously used programs which CAPE-COD provides is the wide variety and flexibility in its computed outputs. These include any branch voltage or current, any node voltage, or any of the following transmission related quantities: insertion gain or loss; voltage, current, or power gain or loss; return loss or reflection coefficient; load impedance or admittance; driving point impedance or admittance; $\mu\beta$ gain and $\mu\beta$ impedances; and Y, Z, or S parameter matrix values.

The $\mu\beta$ gain, i.e., the loop gain of a feedback amplifier, is especially useful for verifying stability in the amplifier. Since many of these outputs are complex numbers, they can be listed in any of the following forms: real and imaginary parts, magnitude and angle (in degrees or radians), dB[$= 20 \log_{10}$(magnitude)] and envelope delay (the derivative of the angle with respect to angular frequency).

Sensitivity studies are facilitated by another feature of CAPE-COD. After an analysis with nominal circuit values has been completed, it is possible to modify the values of any (one or several) element, subject to constraints if desired. A second analysis is then done, and the output quantities can be the *difference* in any normal output that results

from the modification of the circuit element values. This provides a direct measure of circuit sensitivity to any change, large or small, in any element values.

III. OPTIMIZATION

For a system as complex as L5, no exact design procedure exists at any level, either device, circuit, or system. Many iterations are required from an initial to a "final" design. Various tradeoffs are weighed, merit criteria are assigned, and design paramcters are adjusted to yield a result that is optimum under given assumptions.

Various optimization programs, including SUPROX,[2] have been used at Bell Laboratories and have achieved good results. However, as with basic ac analysis programs, shortcomings led to the development of a new general-purpose optimization program named GPOP. Its capabilities are described briefly, and some key applications to L5 design are given as illustrations of its use.

3.1 Program capability

The GPOP is capable of minimizing a criterion function $F(\mathbf{x})$ by adjusting the set of n parameters $x_1, x_2, \cdots, x_n$ (or in vector notation $\mathbf{x}$) which may be bounded (above and/or below), unbounded, or fixed. A simplified program flow chart is shown in Fig. 1. Built into the program is a least-pth error criterion function that has the following form:

$$F(\mathbf{x}) = \sum_{j=1}^{l} \sum_{i=1}^{m} w_{ij}[y_{ij}(\mathbf{x}) - r_{ij}]^p,$$

where

$y_{ij}(\mathbf{x}) = y_j'(\mathbf{x}, a_i)$ is the jth response function value evaluated for the ith independent variable a_i. For example, y_1', y_2', and y_3' may be loss, delay, and input impedance of a given circuit, while $\mathbf{x}$ represents the circuit parameters and a_i is the ith frequency point.

r_{ij} is the requirement of the jth response function for the ith independent variable. If a *range* of requirements is requested instead of single value, and if $y_{ij}(\mathbf{x})$ falls outside the range, the r_{ij} in the formula is the value closer to $y_{ij}(\mathbf{x})$.

p is any positive even integer.

w_{ij} is the weighting constant of the jth response function at the ith requirement point.

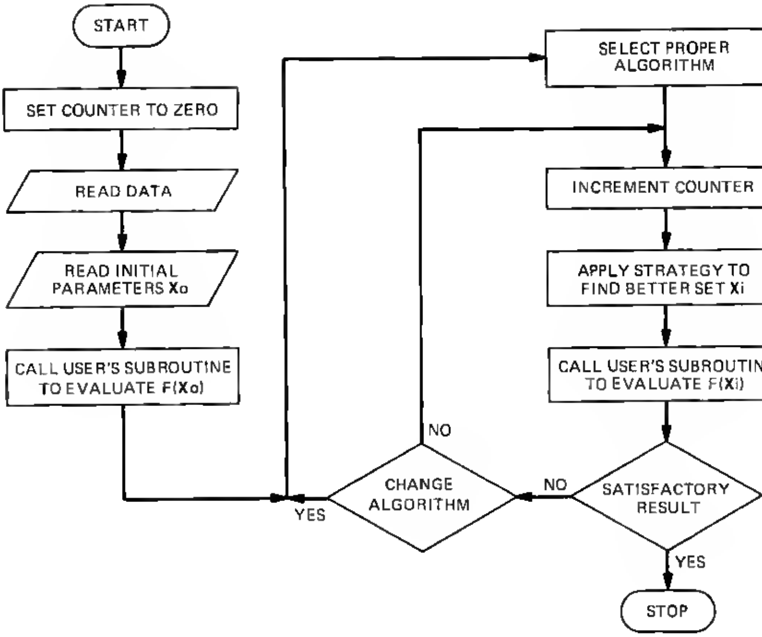

Fig. 1—Simplified flowchart for GPOP.

If the least-$p$tb error criterion is not adequate for an application, the GPOP program also accepts a user-defined criterion function in the form of a subroutine. A user-supplied subroutine must be given to evaluate $y_{ij}(\mathbf{x})$ for the above error criterion. This subroutine is often in the form of a complete ac circuit analysis done by another program, such as CAPE-COD, but now treated as a subroutine.

A variety of strategies or algoritbms exists for finding the optimum. Since the efficiency of these strategies is problem-dependent, it is desirable to have many available from which the user (or program itself) can choose. The GPOP is equipped with Fletcher-Powell,[6] steepest descent,[7] Nelder-Mead,[8] least-pth approximation[9] (least-squares when $p = 2$), and many others. Stopping the search or switching to another algorithm may be done either after a given number of iterations* or after the percent improvement is less than a specified value for five consecutive iterations.

Since the above algorithms work only for unconstrained optimization problems, the optimization of a bounded parameter problem (whicb is

* In GPOP, an iteration corresponds to an improvement; i.e., in one iteration, the algorithm searches until a better criterion value is found.

a simple form of constraint) is accomplished by transforming the bounded parameters into free parameters. A variety of transformations have been built into the program. A user may also supply a specific transformation for a particular application. The gradient of the response function can be computed either from a numerical difference approximation or from a user-supplied expression. Input and/or output subroutines for pre- and/or postoptimization computation add another useful capability. In some problems, it is necessary to apply parameter constraints (such as interrelationships between parameters) independently of any parameter bounds. The input or output routines in GPOP allow the user to apply sequential unconstrained optimization techniques to constrained optimization problems.[10]

3.2 Application

The application of GPOP to a *system* design problem is given in a later section; here, a circuit problem, the optimization of a fixed deviation equalizer, is used for illustration. In practice, the fixed equalizer applies an average correction for the difference between repeater gain shape and cable loss shape.[11,12] Residual errors are then corrected in the field, using adjustable E1-E2 equalizers. The criterion function does not (and should not) consider the error at the output of the fixed deviation equalizer, but rather the final error after all

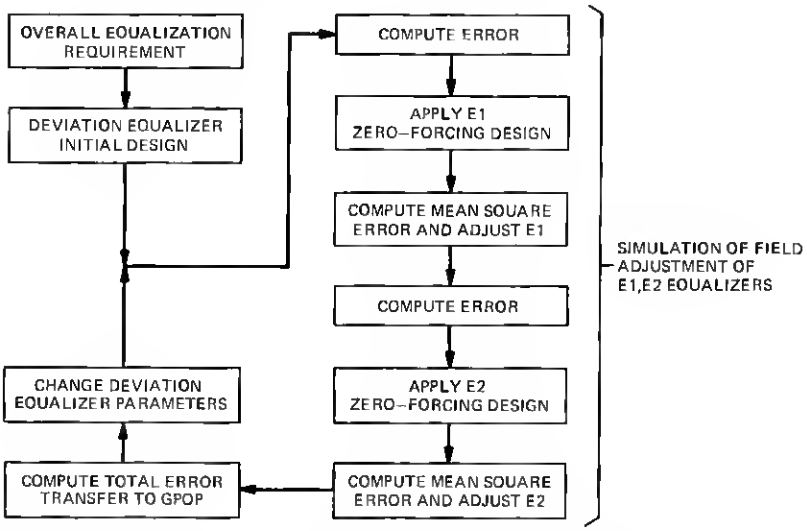

Fig. 2—Flowchart for optimization of a fixed deviation equalizer.

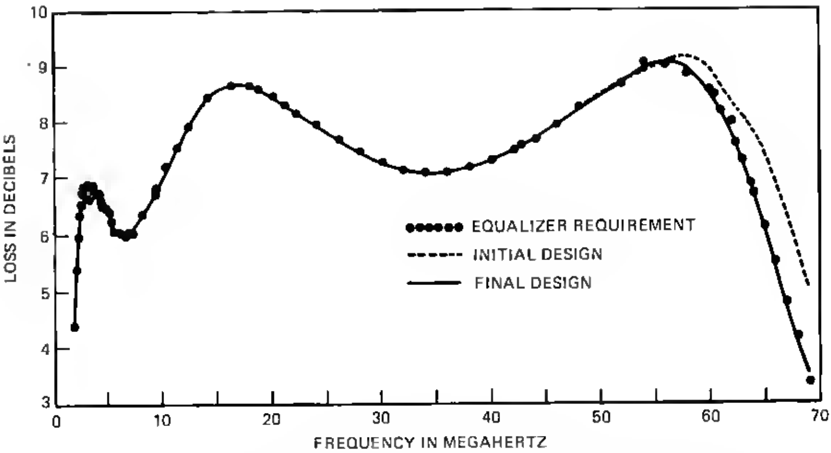

Fig. 3—Performance of fixed deviation equalizer before and after optimization.

three equalizations have been applied. Thus, the program must simulate the field adjustment process before it can optimize the parameters of the fixed equalizer. Figure 2 shows the flow chart for this process, and Fig. 3 shows the result of optimization. Four stages of bridged-T equalizers, having 32 elements, were needed in the final design for the excellent match to requirements.

IV. STATISTICAL TRANSMISSION MEASUREMENTS OF REPEATERS FOR EQUALIZER DESIGN

The equalization hierarchy of the L5 system defined two levels of accuracy needed for transmission measurements on subsystems such as the basic repeater, regulating repeater, and equalizing repeater. These levels must be kept in mind to appreciate the measurement problems that had to be solved. First, the basic repeater provides a gain shape to compensate for cable loss (proportional to $\sqrt{f}$), going from 4.97 to 32.04 dB over the range from 1.6 to 66 MHz. The manufacturing goal was that basic repeater gains would not deviate from target shape by more than ± 0.1 dB at the two-sigma points of the statistical distribution. Thus, a transmission measurement accuracy of about 0.01 dB at each frequency was needed. The wide dynamic range (30 dB) and frequency range (nearly two decades) made this 0.01-dB target difficult to achieve. Since measurements could be made on only a limited number of basic repeaters, but the mean shape had to be extrapolated to apply to many repeaters in the system, the accuracy of individual measurements should, if possible, exceed the 0.01-dB

target. Only a test set such as the COTMS[13] was able to provide this capability with the speed needed for a thorough job. Typical runs included 100 discrete frequencies across the band and took one minute per run.

The second level of measurement accuracy was needed for the remaining steps in the hierarchy, the regulating and equalizing repeaters. Since these provide mainly second- or third-order corrections, the dynamic ranges to be covered were relatively small, but accuracies needed approached 0.001 dB over the same wide frequency range. Nothing can be above suspicion at the 0.001-dB level (i.e., one part in 10^4). Mistermination or mismatch errors are the most frequent and insidious problem. Over this frequency range, even the best available pads needed to mask the test set impedances from those of the unknown showed significant frequency shaping of both transmission and impedance characteristics. Inexpensive mass-produced connectors, as used in field installations, have poor connection repeatability and impedance in terms of these accuracy requirements. But the temptation is strong for the infrequent user to assume that an instrumental resolution of 0.001 dB will guarantee him an absolute accuracy of 0.001 dB in any measurement. So regular accuracy verification of the test equipment, use of selected highest quality test fixtures and masking pads, and careful error analysis of each measurement was essential; when these checks were omitted, serious errors were often discovered later.

A third complication arose from the need for detailed temperature characterization of the repeaters. The maximum temperature coefficient of 0.0016 dB per °F for the basic repeater is a value critical in the design of the dynamic equalizer and the regulating repeater. A computer-controlled environmental test chamber was used with COTMS to accumulate sufficient data to determine this. Continuous control and automatic data logging minimized the operator interaction, but round-the-clock measurements required months to complete. Long interconnecting cables and a network of multiplexing relays introduced serious line reflections, requiring special care in selecting and locating masking pads. When the measurements were self-consistent, a standard statistical analysis program in another computer completed the temperature characterization.

V. DISTORTION ANALYSIS

Nonlinear distortion is a serious problem for a long-haul, frequency-division-multiplexed, analog communications system using solid-state

devices. Most serious are the second- and third-order intermodulation distortions, caused mainly by the transistors. Allowable distortions must be far below fundamental signals in practice; for example, the third-order distortion should not exceed a level 90 dB below fundamental. Calculation of such small distortions requires not only very accurate transistor modeling, but also appropriate mathematical tools to extract such a small fraction of the main signal.

5.1 A new distortion analysis program

Circuits having only a few transistors connected in cascade and a linear feedback path have been analyzed by S. Narayanan,[14] using the Volterra series approach. While answers are obtained in closed form, the computer implementation is not general and cannot handle the large complex circuits in an L5 repeater. So a new program was written to treat the second- and third-order distortions of transistor circuits under small signal conditions. It is called the nonlinear distortion analysis program (NODAP) and is based on the perturbation method.

For this type of problem, successful analysis depends on an accurate active device model. The integral charge control model (ICM) of Gummel and Poon,[15] which contains many nonlinear effects not included in conventional models, was chosen. Some more important effects include conductivity modulation, base push out, Early voltage, and avalanche effects. Expanding the ICM equations around a given bias point and expressing terminal currents in terms of junction voltages leads to an appropriate small-signal model. This is readily separated into linear and nonlinear parts, the linear part being representable by a hybrid-π equivalent circuit, or simply a two-port admittance matrix $Y(j\omega)$ (Fig. 4). Two controlled current sources, i_{Ne} and i_{Nc}, represent the nonlinear part. These together represent the intrinsic transistor, to which must be added appropriate parasitic elements; the parasitics are treated linearly except for the inactive collector capacitance.

The perturbation method works well (two iterations are sufficient), since the nonlinearities are small for small-signal conditions. The computational algorithm[16] uses any linear circuit analysis program (in this case, CAPE-COD). The result is an accurate, efficient, general-purpose program. The only nonlinear elements in the circuit are i_{Ne} and i_{Nc}, and they can be shown to be equivalent to two distortion current generators having amplitudes and phases determined by the second- and third-order nonlinear coefficients and the linear characteristics of the transistor circuit. This approach for a single transistor

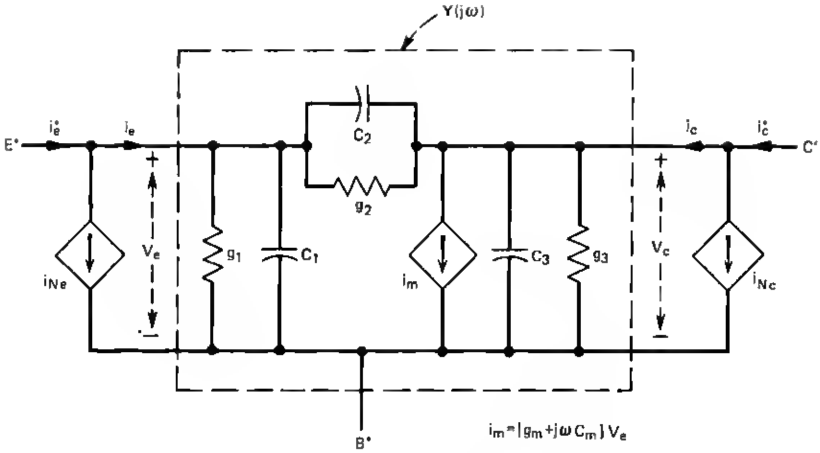

Fig. 4—Small-signal nonlinear transistor model.

can be readily extended to multitransistor circuits, since the distortion computation is carried out by a quasilinear approach and superposition holds.

The sequence of calculations in NODAP is interesting, showing how major programs can be combined. Given the circuit topology with the ICM parameters, bias point of each transistor, and sets of three fundamental frequencies (f_1, f_2, and f_3), NODAP computes the linear and nonlinear coefficients for the transistor model. The linear characteristics are passed to CAPE-COD to calculate output voltages, load admittance, and junction voltages. These are returned to NODAP to calculate the second-order intermodulation current sources at $f_1 \pm f_2$, $f_1 \pm f_3$, and $f_2 \pm f_3$. These sources go back to CAPE-COD to calculate second-order distortion voltages. This type of looping is similarly repeated for the third-order distortion. The final step, by NODAP, prints out the calculated second- and third-order distortion indices M_{2E} and M_{3E}.

A "linear" and "nonlinear" option in NODAP allows the "turning off" or "turning on" of the distortion sources in any stage in an amplifier, to determine the contributions from that stage. The distortion effects of various nonlinearities can thus be isolated, giving insight impossible from direct measurements alone. Also, distortion *vectors* are computed, allowing the design of distortion cancellation circuits.

The calculated results for several L5 circuits have been in very good agreement with measured data, within 2 dB over a wide range of bias

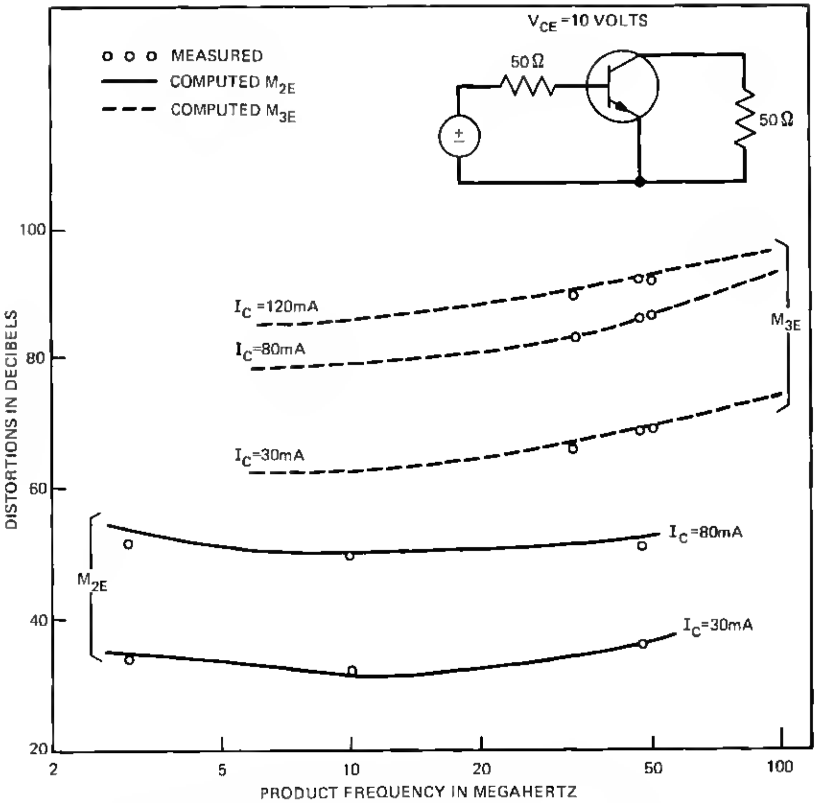

Fig. 5—Distortions of a common-emitter transistor amplifier.

conditions and frequencies. Figure 5 shows a comparison of predictions and measurements over the L5 frequency band.

5.2 Device modeling for distortion analysis

Accurate device modeling is essential to the success of NODAP in predicting nonlinear distortion. A program called extraction of parameters for the ICM of transistors (EPIC) has been developed to extract 30 ICM parameters from three sets of measured data. The raw data include the dc V-I characteristics, the junction capacitance characteristics, and the small-signal ac characteristics measured over a wide range of bias conditions. The previously described GPOP program is used to obtain the best ICM parameters by optimizing the match to the data. Those parameters associated with a single data set are matched first,

progressing to parameters that affect multiple data sets. This gives accurate results (physically realistic) quickly.

The main L5 transistor parameters were carefully extracted, using EPIC. In addition to their usefulness in predicting distortion, the linear model parameters were used in CAPE-COD for normal ac circuit analysis, giving excellent results. Agreement between computed and measured S parameters for the transistor was within 2 dB over the entire L5 frequency range.

VI. COMPONENT CHARACTERIZATION IN TERMS OF *S* OR *Z* PARAMETERS

Almost all the electronic subassemblies are built with components that are treated as classical "lumped elements" such as resistors, capacitors, inductors, or diodes, or as multiport devices such as transformers or transistors. Near the top L5 frequencies (66 MHz), the lumped element approximation is often inadequate and transmission line delays become important. Small capacitances and inductances, normally considered "parasitics," may have substantial effects in a circuit, especially over the extreme bandwidths covered by feedback amplifiers.

Simple passive components, such as two-terminal impedances, were measured automatically over frequency ranges up to 1000 MHz. Not only was the variation of element value with frequency obtained (caused by inadequate models), but derived properties were also obtained, such as the permeability and loss of magnetic materials, "Q" variation with frequency, self-resonant frequencies of reactive components, and linear two-element equivalent circuits of diodes. Such measurements were made quickly and very accurately on the COTMS, using an impedance calculating program. Impedances from milliohms to megohms can be measured over the range from audio to microwaves. Accuracies of the order of 0.1 percent are realized for real and imaginary components between 1 and 1000 ohms. A recent addition to the measurement tools is a computer-operated impedance bridge (COZY) that provides even greater resolution (approaching 10 ppm) and impedance range, though at frequencies limited to below 30 MHz.

The characterization of transformers and transistors as multiport devices was at a more sophisticated level. Basic measurements of voltage scattering (S) parameters were made automatically on COTMS, using a special program and calibration technique which compensates for the mismatch errors caused by imperfect test set impedances.[17] These corrections are especially important since test set impedances do not remain constant over wide frequency ranges. From the basic S

parameters, other matrix parameters such as H or Y were obtained by computer data transformation. The H parameters are particularly useful for transistors, while the Y parameters fit naturally into circuit analysis programs based on nodal techniques. The multiport transformers used in the L5 amplifiers were characterized by merging several sets of two-port measurements into a 4-by-4 Y matrix representation.

Many of these characterization jobs were not carried out until inexplicable problems appeared during the development of circuits. However, with the presently available tools that are quick, accurate, and easy to use, all critical component types can be characterized automatically before use in model circuits. Such routine checks can prevent many problems and save considerable time in the long run. A component as simple as a resistor has many nanohenries of body and lead inductance that can play havoc in high-frequency circuits, yet it takes only a few minutes to characterize the component with modern tools.

VII. SYSTEM DESIGN

In repeatered analog transmission system design, the optimal allocation of total system noise caused by various sources and subject to certain constraints is an important but tedious task, if the conventional approach is used. Manual trial-and-error modification of transmission levels and calculation of the resulting noise at various system points are time-consuming processes. All these are interdependent functions of repeater gain, noise figure, modulation coefficients, etc. A nonoptimum assignment may lead to failure to meet system design objectives and to possible redesign of basic elements, starting a new system design cycle. Since the process requires optimizing a performance function subject to a given set of system constraints, modern optimization techniques are applicable.

7.1 Computation of intermodulation noise

An intermodulation noise computation program called NEWMOD was written at Bell Laboratories in the 1960's for the development of submarine cable systems. This program employs a modified version of Bennett's product count method[18] in a formulation that takes into account the effect of the nonlinear phase characteristic of repeaters. The NEWMOD program was used extensively in the development of the L5 system, in which the introduction of phase shaping networks effectively randomized the addition of third-order intermodulation prod-

ucts. This allowed the channel capacity of the system to be increased by about 20 percent for the same noise performance.

The product count method requires too much computer storage and processing time to allow the coupling of NEWMOD with GPOP for system noise optimization. This difficulty can be avoided by modifying the earlier formulation so that a high-order Chebyshev numerical-integration formula is employed to replace the product count approach. Based on this new formulation, a program named noise optimization for repeatered analog transmission systems (NORATS) has been implemented and is briefly described.

7.2 NORATS program objective and organization

The main objective of this optimization is to select the best repeater output transmission level as a function of frequency for the given conditions. For a noise-limited system such as L5, the total system noise should be minimized. For an overload limited system, the average power output of repeaters should be minimized so that the total system noise still satisfies the requirements. In both problems, we may consider the modulation coefficients as additional design parameters subject to certain limitations. All these problems can be considered special cases of minimizing the following criterion function

$$F(\mathbf{x}) = \sum_{i=1}^{m} w_i[y_i(\mathbf{x}) - r_i]^p,$$

where

$y_i(\mathbf{x})$ = total system noise (in dBrnc0) at frequency f_i $i = 1, 2, \cdots, m-3$,

$y_{m-2}(\mathbf{x})$ = average value of the second-order modulation coefficient (in dB),

$y_{m-1}(\mathbf{x})$ = average value of the third-order modulation coefficient (in dB),

$y_m(\mathbf{x})$ = equivalent rms power of a single tone at repeater output (in dBm),

r_i and w_i are the corresponding requirements and weighting constants, p is a positive even integer, and the components of $\mathbf{x}$ are the coefficients of the polynomial representations of the transmission level and modulation coefficients as functions of frequency. The total system noise function in the equation is the power addition of thermal, second-order,

and third-order intermodulation noise. This criterion function is in a form ready to be used in the GPOP program described in Section II.

7.3 NORATS applications

Solutions to various design problems can be carried out using NORATS by choosing appropriate weighting constants in the criterion function. For a noise-limited system, the weighting constant w_m corresponding to the output power is set to zero, since this allows the optimum transmission level to be determined by minimization of total system noise, regardless of output power. A relatively large w_m corresponds to an overload-limited system. The weighting constants w_{m-1} and/or w_{m-2} are set to zero if the corresponding modulation coefficients are given instead of being treated as design parameters. Recent NORATS runs have confirmed that earlier designs approached an optimum.

VIII. EQUALIZER CHARACTERIZATION AND ALIGNMENT

The strategy of using distributed corrections for the separate temperature variations of the cable and the repeaters (regulation), and for deviations from nominal frequency shaping caused by manufacturing tolerances (equalization), is a complex one. The strategy depends on accurate characterization of the various equalizer and regulator circuits to be effective, and this was considerably simplified by the use of COTMS. One special program for alignment during manufacture deserves description. It illustrates that, in some problems, even the most sophisticated measurement equipment alone is not enough to fill the need. By incorporating within the computer-controlled test set an algorithm for adjusting a set of nonorthogonal controls, considerable time and cost savings are achieved.

The 4211A network generates a $\sqrt{f}$ shape for use in regulating the L5 system against changes in cable loss with temperature.[12] Six adjustable elements, which are not independent, are needed to match the resulting shape to requirements. While their individual effects are readily measured, the interdependence makes final adjustment during manufacture nearly impossible by trial-and-error methods. The problem could be solved on a separate computer, but this would require voluminous data transfer and lost time. The logical time and place to carry out the adjustment were simultaneously with the measurement process. The details of the procedure and its effectiveness are described in another article in this issue.[12] This particular combination of precision measurement, data reduction, computer prediction, and output

"instructions" to an operator, all done in a single machine, will he the way of the future for such complex tasks.

IX. CONCLUSIONS

The complex design prohlems to he solved in the development of the L5 system led to the writing of a series of new computer programs that are highly interrelated and yet very useful independently. CAPE-COD provides ac circuit analysis with flexihle input and output, speed of computation, and the ahility to handle large circuits with very few limitations of types of elements. Continued development is under way to extend its range of application into the microwave region and to improve its interactive and statistical capahilities. The GPOP program implements modern optimization techniques through a variety of algorithms and considerahle flexihility in types of constraints allowed. Both GPOP and CAPE-COD contain links to simplify their comhined use, making an extremely flexihle combination. The NODAP introduces a new method of handling nonlinear distortion prohlems, working together with CAPE-COD. Input data for NODAP are provided hy EPIC, a transistor model parameter extraction program that works together with GPOP. A new system design program, NORATS, also works with GPOP to optimize the noise performance of repeatered analog transmission systems.

The usefulness of computer-operated transmission measurement equipment was enhanced hy the development of advanced impedance measurement and modeling programs. Automatic measurement of many repeaters under computer-controlled environmental conditions was facilitated, along with data links to transfer measurement and characterization data directly to another computer where they could be used in CAPE-CON. A specialized alignment program was written for use in COTMS.

While each of these new tools is important in its own right, their comhination and coordinated development provided an extremely powerful set of tools. Clearly, the L5 project could not have heen completed on schedule without these computer aids to design and measurement.

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